Simulation and Traffic Routing for Efficient Allocation and Management

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Abstract

STREAM - Simulation and Traffic Routing for Efficient Allocation and Management is a simple simulation tool that I have developed as a part of my project work over the summer. It is intended to locate the importance of Braess's Paradox in effectively allocating resources over networks where there is a flow of traffic. It is just a simple tool to analyse if there is any extraneous edge in the network and whether the functioning efficiency of the entire network would improve if it is not present.

Braess's Paradox

"Braess's paradox is the observation that adding one or more roads to a road network can slow down overall traffic flow through it. The paradox was first discovered by Arthur Pigou in 1920."

A very important discovery in the domain of game theory, it provides a practical problem that is posed to networks with high traffic flow and limited resources or components. It gives an outlook towards the design of such networks and provides intuition on how more efficiency can be achieved in the overall network. Our common belief and intuition tells us that addition of more components in a network would generally tend to ease the pressure on them and make it more efficient overall, but that doesn't seem to hold true in all cases. It is observed that in quite a few places, removal/ reduction of certain components in the network tend to lead to greater overall efficiency in the network.

The concept of Nash Equilibrium helps us to understand the counter intuitive behaviour of networks. Braess's Paradox is considered to be a game with a pure strategy Nash Equilibrium. This can be described in the following way,

Given a strategic form game $\Gamma = \langle N, (S_i), (U_i) \rangle$ *, the strategy profile* $s = (s_1, s_2, ..., s_n)$ *is a pure strategy Nash equilibrium of* Γ *if, the utilities payoff of the given strategy,*

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \quad \forall\, i = (1, 2, 3..., n)$$

That is, each player's Nash equilibrium strategy is a best response to the Nash equilibrium strategies of the other players.

STREAM

This is a tool that simulates the Braess's paradox on simple networks and analyses to check if the efficiency of the network given a traffic profile is most suitable in the network. The main idea is to perturb a flow network with a single user, and then check if Braess's paradox is followed or not. First the network is set up with the given map and initial flow values. The weights to the edges are set up in such a way that they are affected by the number of users on that particular edge. Here, only a linear dependence is assumed to be followed for each edge. This can be proven to be a suitable assumption as Braess's paradox can be shown to work in any case where there is a functional dependence on the number of people on that particular edge in the network.

Since we are unsure of which of the edges in the network cause the paradoxical behaviour, this tool loops through each of the directed edge in the network and does the following things in the analysis:

- Remove/ restrict the given edge and then find the shortest path for the trip using Dijkstra's algorithm.
- Add a non weighted edge between two points and then simulate the shortest path after redistribution of the traffic on the other edges.

Although this does not take into account all the possible cases that are possible, it provides a ground level view of how this network can be affected by small changes. If the traffic over a small component of the graph is to be analysed, we may use this to perturb small changes in the network over that area and check for the efficiency of the network.

Few of the assumptions taken into account in the following project:

- There exists a steady flow of traffic in the edges and perturbation of the network does not greatly influence the other flows. This can be justified as in this report, we are tracking very small perturbations on a small scale and thus the other equilibrium flows should not be affected.
- We can figure out the Nash Equilibrium of the traffic flow by providing a greedy/selfish strategy for all the users. The users even in a generalised practical case would choose the path that is shortest/least expensive for them. This can be proven as follows,

  Consider any other strategy profile for the user of the network, $s = (s_1, s_2, s_3..., s_n)$. *It can then be shown that if any of the user does not follow the shortest/least expensive path,* $u(s_1, s_2, s_3..., s_n)$ *will always be* $\leq u(s_i^*, s_{-i}^*)$ as mentioned above to be the equilibrium condition. Thus any such strategy will be a pure strategy Nash Equilibrium in the network.
- There are certain types of edges/paths in the network with linear dependence of their weights on the number of users on that particular edge at a given time. Braess's paradox can be shown to be working on such edges with linear dependence only.

Functioning:

This is a brief description of the idea behind how the tool is meant to work and simulate the network flows and perturbations:

- Using the data, we first form the graph and network by adding the nodes and edges. Then we classify the edges into multiple types by their linear factors. The weights of each of the edges are given by:

$$weight = base\ weight + linear\ factor * number\ of\ users$$

- After the formation of the graph, and adding the flows on each of the edge, we perturb the network by introducing a user. This user would then perturb the network by wanting to travel from a source to the destination.

- The two things as mentioned previously are done once the new user is added. Firstly the shortest path for the user from the source to the destination is calculated by using Dijkstra's algorithm. Then we perform the following procedures, first we remove each edge of the graph and then simulate the same thing, by redistributing the traffic on the removed edge. We check if this would in any case improve the shortest path from the source to the destination. If such a case is found, then it would imply Braess's Paradox is being followed over here.

- Next, similarly we add a non weighted edge between every pair of edges other than the source and the destination. In an ideal case scenario, this should decrease the total cost of the path. However due to the redistribution and formation of the new Nash Equilibrium, we may find it to have a worser shortest path than the original one. This is a clear indication of Braess's paradox being valid.

Thus in these ways we predict whether the allocation of the resources in a network is in the most efficient way possible or not. This is not the best analysis but for small networks we can loop through all the possibilities in this way and check if the allocation is good enough.

Analysis

After simulating using the above format with a small map of my university and some other randomizations (which were somewhat forced to achieve the results), some things to notice were:

- Most of the occurrences of this paradox were in junctions of a similar structure. They involved two nodes that had two alternate pathways of almost symmetric structure. It may involve some other intermediate nodes as well but most of the times, it was two symmetric nodes in the intermediate pathways. The paradox occurred when there was an introduction of another intermediate pathway providing an alternate path between few of the intermediate nodes. An explanation that was possible was that the Nash Equilibrium was set up initially in a symmetric manner and even a small perturbation caused it to have a great effect on the imbalance it caused. This could be attributed to the fact that the functional (linear in this case) dependence causes the redistribution in such a way that the additional intermediate path between the nodes causes a dilemma in choosing the shortest path and since everything is assumed to be working in a greedy/selfish way it causes a dead-block and overall cost increases for each of the users. This ideally would have atleast reduced the cost for a few of the users but since each user is treated symmetrically, there occurs an issue in which the path cost for everyone is increased.

- Another noticeable pattern was that the paradox mostly occurred in the second process in which we introduced new edges between every pair of nodes one by one. So the paradox is mostly expected to occur when there is an addition of a new resource. Although it can be forcefully made to occur by removing an edge in a network, this is a rare case and according to the limited data that has been analysed so far, it is more likely to occur in the former case.

These were my observations on this topic by analysing a few small maps. This may not be an extensive case study but does provide some insights on how the paradox affects the design of networks involving distribution of resources. More changes to the tool are to be added and any new insights/contributions to the same are welcome!

Bibliography

The above project was done under the guidance of Prof. Y Narahari, Indian Institute of Science. The main ideas and information has been taken from the book authored by him - Game Theory and Mechanism Design

Other sources of the project include:

- Wikipedia
- Harvard Edu
- Paper